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8. The sum is evaluated by recognising that it is a geometric progression with common ratio $e^{2i\pi/n}$ which may be summed using the standard formula and as $1 - e^{2i\pi/n} \neq 0$, the denominator is non-zero so the sum is zero. By simple trigonometry, $s = d - r\cos\theta$. As $r = ks$, $r = \frac{kd}{1+k\cos\theta}$. Thus $l_j = \frac{kd}{1+k\cos\theta} + \frac{kd}{1+k\cos(\theta+\pi)}$ where $\theta = \alpha + (j-1)\pi/n$. Simplifying, $l_j = \frac{2kd}{1-k^2\cos^2\theta}$. The summation of the reciprocals of this expression is simply found using a double angle formula and then by expressing the trigonometric terms as the real part of the sum at the start of the question.