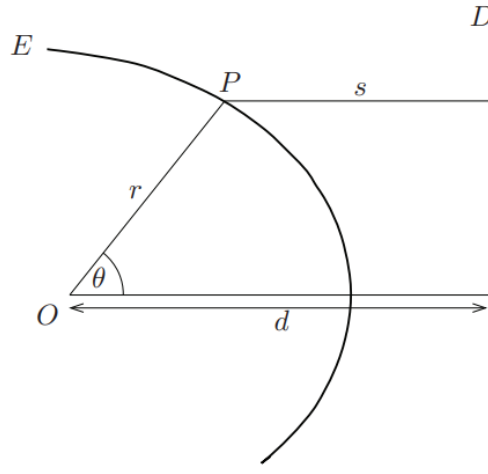


## STEP III, 2013 , Q8

- 8 Evaluate  $\sum_{r=0}^{n-1} e^{2i(\alpha+r\pi/n)}$  where  $\alpha$  is a fixed angle and  $n \geq 2$ .

The fixed point  $O$  is a distance  $d$  from a fixed line  $D$ . For any point  $P$ , let  $s$  be the distance from  $P$  to  $D$  and let  $r$  be the distance from  $P$  to  $O$ . Write down an expression for  $s$  in terms of  $d$ ,  $r$  and the angle  $\theta$ , where  $\theta$  is as shown in the diagram below.



The curve  $E$  shown in the diagram is such that, for any point  $P$  on  $E$ , the relation  $r = ks$  holds, where  $k$  is a fixed number with  $0 < k < 1$ .

Each of the  $n$  lines  $L_1, \dots, L_n$  passes through  $O$  and the angle between adjacent lines is  $\frac{\pi}{n}$ . The line  $L_j$  ( $j = 1, \dots, n$ ) intersects  $E$  in two points forming a chord of length  $l_j$ . Show that, for  $n \geq 2$ ,

$$\sum_{j=1}^n \frac{1}{l_j} = \frac{(2 - k^2)n}{4kd}.$$



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