

STEP III, 2013 , Q7

- 7 (i) Let $y(x)$ be a solution of the differential equation $\frac{d^2y}{dx^2} + y^3 = 0$ with $y = 1$ and $\frac{dy}{dx} = 0$ at $x = 0$, and let

$$E(x) = \left(\frac{dy}{dx}\right)^2 + \frac{1}{2}y^4.$$

Show by differentiation that E is constant and deduce that $|y(x)| \leq 1$ for all x .

- (ii) Let $v(x)$ be a solution of the differential equation $\frac{d^2v}{dx^2} + x\frac{dv}{dx} + \sinh v = 0$ with $v = \ln 3$ and $\frac{dv}{dx} = 0$ at $x = 0$, and let

$$E(x) = \left(\frac{dv}{dx}\right)^2 + 2 \cosh v.$$

Show that $\frac{dE}{dx} \leq 0$ for $x \geq 0$ and deduce that $\cosh v(x) \leq \frac{5}{3}$ for $x \geq 0$.

- (iii) Let $w(x)$ be a solution of the differential equation

$$\frac{d^2w}{dx^2} + (5 \cosh x - 4 \sinh x - 3)\frac{dw}{dx} + (w \cosh w + 2 \sinh w) = 0$$

with $\frac{dw}{dx} = \frac{1}{\sqrt{2}}$ and $w = 0$ at $x = 0$. Show that $\cosh w(x) \leq \frac{5}{4}$ for $x \geq 0$.



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