

## STEP III, 2013 , Q6

- 6 Let  $z$  and  $w$  be complex numbers. Use a diagram to show that  $|z - w| \leq |z| + |w|$ .

For any complex numbers  $z$  and  $w$ ,  $E$  is defined by

$$E = zw^* + z^*w + 2|zw|.$$

- (i) Show that  $|z - w|^2 = (|z| + |w|)^2 - E$ , and deduce that  $E$  is real and non-negative.
- (ii) Show that  $|1 - zw^*|^2 = (1 + |zw|)^2 - E$ .

Hence show that, if both  $|z| > 1$  and  $|w| > 1$ , then

$$\frac{|z - w|}{|1 - zw^*|} \leq \frac{|z| + |w|}{1 + |zw|}.$$

Does this inequality also hold if both  $|z| < 1$  and  $|w| < 1$ ?



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