

## STEP III, 2013 , Q4 MS

4. The initial result is obtained by expanding the brackets and expressing the exponentials in trigonometric form. The  $(2n)$ th roots of  $-1$  are  $e^{i\frac{2m+1}{2n}\pi}$ ,  $-n \leq m \leq n-1$ , which lead to the factors of  $z^{2n} + 1$  and these paired using the initial result give the required result. Part (i) follows directly from substituting  $z = i$  in the previous result, and as  $n$  is even,  $z^{2n} + 1 = 2$ . Using the given factorisation in part (ii), the general result can be simplified by the factor

$z^2 - 2z \cos \frac{n}{2n} \pi + 1 = z^2 + 1$ . Again substituting  $z = i$ , and that  $\cos \frac{2n-r}{2n} \pi = -\cos \frac{r}{2n} \pi$  gives the evaluation required.



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