

STEP III, 2013 , Q3 MS

3. The scalar product of p_i with $\sum p_r$, which is of course zero, can be expanded giving $p_i \cdot p_i = 1$ and three products $p_i \cdot p_j$ which are equal by symmetry, giving the required result. Expanding the expression suggested in (i), gives $\sum_{i=1}^4 (p_i \cdot p_i - 2x \cdot p_i + x \cdot x)$, which, bearing in mind that $p_i \cdot p_i = 1$, $x \cdot x = 1$, and that $x \cdot \sum_{i=1}^4 p_i = 0$, gives the correct result. Considering that $p_1 \cdot p_2 = -\frac{1}{3}$, $p_2 \cdot p_2 = 1$, and that a is positive, enables the given values to be found. Similarly $p_1 \cdot p_3 = -\frac{1}{3}$, $p_2 \cdot p_3 = -\frac{1}{3}$, and $p_3 \cdot p_3 = 1$ yields $P_3, P_4 = \left(-\frac{\sqrt{2}}{3}, \pm \frac{\sqrt{2}}{\sqrt{3}}, -\frac{1}{3}\right)$. In (iii), using the logic of (i), $(XP_i)^4 = ((p_i - x) \cdot (p_i - x))^2 = 4(1 - x \cdot p_i)^2$, as required. Expanding this, and using the coordinates of X and those of P_i that have been found,

$$\begin{aligned} \sum_{i=1}^4 (XP_i)^4 &= 16 + 4 \left(z^2 + \left(\frac{2\sqrt{2}}{3}x - \frac{1}{3}z \right)^2 + \left(-\frac{\sqrt{2}}{3}x + \frac{\sqrt{2}}{\sqrt{3}}y - \frac{1}{3}z \right)^2 + \left(-\frac{\sqrt{2}}{3}x - \frac{\sqrt{2}}{\sqrt{3}}y - \frac{1}{3}z \right)^2 \right) \\ &= 16 + 4 \left(\frac{4}{3}x^2 + \frac{4}{3}y^2 + \frac{4}{3}z^2 \right) = \frac{64}{3} \text{ which is sufficient.} \end{aligned}$$



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