

STEP III, 2013 , Q2

- 2 In this question, you may ignore questions of convergence.

Let $y = \frac{\arcsin x}{\sqrt{1-x^2}}$. Show that

$$(1-x^2)\frac{dy}{dx} - xy - 1 = 0$$

and prove that, for any positive integer n ,

$$(1-x^2)\frac{d^{n+2}y}{dx^{n+2}} - (2n+3)x\frac{d^{n+1}y}{dx^{n+1}} - (n+1)^2\frac{d^ny}{dx^n} = 0.$$

Hence obtain the Maclaurin series for $\frac{\arcsin x}{\sqrt{1-x^2}}$, giving the general term for odd and for even powers of x .

Evaluate the infinite sum

$$1 + \frac{1}{3!} + \frac{2^2}{5!} + \frac{2^2 \times 3^2}{7!} + \dots + \frac{2^2 \times 3^2 \times \dots \times n^2}{(2n+1)!} + \dots$$



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