

## STEP III, 2013 , Q13 MS

13. integrating  $0 \leq f(t) \leq M$  between limits of 0 and  $x$  gives the result of (a) (i), and integrating the left hand side by parts yields part (ii). As  $kF(y)f(y)$  is a probability density function,  $\int_0^1 k F(y)f(y)dy = 1$ , which can be evaluated using the result of (a) (ii) with  $2g(x) = k$  and so  $k = 2$ .  $E(Y^n) = \int_0^1 y^n 2F(y)f(y)dy \leq \int_0^1 y^n 2Myf(y)dy = 2M\mu_{n+1}$  and using (a) (ii),  $E(Y^n) = \int_0^1 y^n 2F(y)f(y)dy = 1 - n \int_0^1 y^{n-1}(F(y))^2 dy$ , as  $\int_0^1 y^{n-1}(F(y))^2 dy \leq \int_0^1 y^{n-1}My F(y)dy = M \int_0^1 y^n F(y)dy$ , integration by parts gives  $\int_0^1 y^n F(y)dy = \frac{1}{n+1} - \frac{1}{n+1}\mu_{n+1}$ . Part (iii) is derived from part (ii) by rearranging  $1 + \frac{nM}{n+1}\mu_{n+1} - \frac{nM}{n+1} \leq 2M\mu_{n+1}$  and making  $\mu_{n+1}$  the subject, then translating  $n + 1$  to  $n$ .



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