

## STEP III, 2013 , Q13

- 13 (a) The continuous random variable  $X$  satisfies  $0 \leq X \leq 1$ , and has probability density function  $f(x)$  and cumulative distribution function  $F(x)$ . The greatest value of  $f(x)$  is  $M$ , so that  $0 \leq f(x) \leq M$ .

(i) Show that  $0 \leq F(x) \leq Mx$  for  $0 \leq x \leq 1$ .

(ii) For any function  $g(x)$ , show that

$$\int_0^1 2g(x)F(x)f(x)dx = g(1) - \int_0^1 g'(x)(F(x))^2 dx.$$

- (b) The continuous random variable  $Y$  satisfies  $0 \leq Y \leq 1$ , and has probability density function  $kF(y)f(y)$ , where  $f$  and  $F$  are as above.

(i) Determine the value of the constant  $k$ .

(ii) Show that

$$1 + \frac{nM}{n+1}\mu_{n+1} - \frac{nM}{n+1} \leq E(Y^n) \leq 2M\mu_{n+1},$$

where  $\mu_{n+1} = E(X^{n+1})$  and  $n \geq 0$ .

(iii) Hence show that, for  $n \geq 1$ ,

$$\mu_n \geq \frac{n}{(n+1)M} - \frac{n-1}{n+1}.$$



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