

STEP III, 2013 , Q12

- 12** A list consists only of letters A and B arranged in a row. In the list, there are a letter A s and b letter B s, where $a \geq 2$ and $b \geq 2$, and $a + b = n$. Each possible ordering of the letters is equally probable. The random variable X_1 is defined by

$$X_1 = \begin{cases} 1 & \text{if the first letter in the row is } A; \\ 0 & \text{otherwise.} \end{cases}$$

The random variables X_k ($2 \leq k \leq n$) are defined by

$$X_k = \begin{cases} 1 & \text{if the } (k-1)\text{th letter is } B \text{ and the } k\text{th is } A; \\ 0 & \text{otherwise.} \end{cases}$$

The random variable S is defined by $S = \sum_{i=1}^n X_i$.

- (i) Find expressions for $E(X_i)$, distinguishing between the cases $i = 1$ and $i \neq 1$, and show that $E(S) = \frac{a(b+1)}{n}$.

(ii) Show that:

(a) for $j \geq 3$, $E(X_1 X_j) = \frac{a(a-1)b}{n(n-1)(n-2)}$;

(b) $\sum_{i=2}^{n-2} \left(\sum_{j=i+2}^n E(X_i X_j) \right) = \frac{a(a-1)b(b-1)}{2n(n-1)}$;

(c) $\text{Var}(S) = \frac{a(a-1)b(b+1)}{n^2(n-1)}$.



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