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11. As the distance from the vertex to the centre of the equilateral triangle is a, the extended length of each spring is $\frac{a}{\cos\theta}$ giving the tension in each as $kmg\frac{\left(\frac{a}{\cos\theta}-a\right)}{a}$ which simplifies to the given result. Resolving vertically $3T\sin\theta = 3mg$, and using the result for T, substituting $\theta = \frac{\pi}{6}$, and rationalising the denominator gives the required value for k. Conserving energy, when $\theta = \frac{\pi}{3}$, gravitational potential energy is $-3mga\tan\frac{\pi}{3}$, elastic potential energy is $\frac{3}{2}kmg\frac{\left(\frac{a}{\cos\frac{\pi}{3}}-a\right)^2}{a} = \frac{3}{2}kmga\left(\frac{1}{\cos\frac{\pi}{3}}-1\right)^2$, whereas when $\theta = \frac{\pi}{6}$, gravitational potential energy is $-3mga\tan\frac{\pi}{6}$, elastic potential energy is $\frac{3}{2}kmga\left(\frac{1}{\cos\frac{\pi}{6}}-1\right)^2$, and kinetic energy is $\frac{3}{2}mV^2$ hence giving V^2.