STEP III, 2013 , Q11

11 An equilateral triangle, comprising three light rods each of length $\sqrt{3}a$, has a particle of mass m attached to each of its vertices. The triangle is suspended horizontally from a point vertically above its centre by three identical springs, so that the springs and rods form a tetrahedron. Each spring has natural length a and modulus of elasticity kmg, and is light. Show that when the springs make an angle θ with the horizontal the tension in each spring is

$$\frac{kmg(1-\cos\theta)}{\cos\theta}\,.$$

Given that the triangle is in equilibrium when $\theta = \frac{1}{6}\pi$, show that $k = 4\sqrt{3}+6$.

The triangle is released from rest from the position at which $\theta = \frac{1}{3}\pi$. Show that when it passes through the equilibrium position its speed V satisfies

$$V^2 = \frac{4ag}{3}(6+\sqrt{3})\,.$$