

STEP III, 2013 , Q1

- 1 Given that $t = \tan \frac{1}{2}x$, show that $\frac{dt}{dx} = \frac{1}{2}(1 + t^2)$ and $\sin x = \frac{2t}{1 + t^2}$.

Hence show that

$$\int_0^{\frac{1}{2}\pi} \frac{1}{1 + a \sin x} dx = \frac{2}{\sqrt{1 - a^2}} \arctan \frac{\sqrt{1 - a}}{\sqrt{1 + a}} \quad (0 < a < 1).$$

Let

$$I_n = \int_0^{\frac{1}{2}\pi} \frac{\sin^n x}{2 + \sin x} dx \quad (n \geq 0).$$

By considering $I_{n+1} + 2I_n$, or otherwise, evaluate I_3 .



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