STEP III, 2012 Q8 MS

8. $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$ so both expressions in part (i) equal -1. Considering either $[(F_nF_{n+3} - F_{n+1}F_{n+2}) - (F_{n-2}F_{n+1} - F_{n-1}F_n)]$ or $(F_nF_{n+3} - F_{n+1}F_{n+2}) + (F_{n-1}F_{n+2} - F_nF_{n+1})$, and applying the recurrence relation, both can be found to be zero. \so the given expression is shown to be 1 if *n* is odd and -1 if *n* is even. The tan compound angle formula enables the proof in part (iii) to be completed once the initial recurrence relation and the result from part (ii) have been applied to the expression obtained following algebraic simplification. Rearranging the result and substituting into the required sum gives, by the method of differences, $\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{F_{2r+1}}\right) = \frac{\pi}{4}$

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