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7. The second order differential equation $\ddot{y} + 7\dot{y} + 6y = 0$ may be obtained by differentiating the first equation, then substituting for $\dot{z}$ using the other equation, and then doing likewise for z using the first again. The solutions for y and z follow in the usual manner. Parts (i) and (ii) yield $z_1(t) = 2e^{-t} - 2e^{-6t}$ and $z_2(t) = \frac{c(e^6-1)}{(e^5-1)}e^{-t} - \frac{ce^5(e-1)}{(e^5-1)}e^{-6t}$ respectively. Part (iii) merely requires the sum to be expanded and two geometric progressions emerge so that $c = \frac{2e}{e-1}\frac{(e^5-1)}{(e^6-1)}$.