

## STEP III, 2012 Q7

- 7 A pain-killing drug is injected into the bloodstream. It then diffuses into the brain, where it is absorbed. The quantities at time  $t$  of the drug in the blood and the brain respectively are  $y(t)$  and  $z(t)$ . These satisfy

$$\dot{y} = -2(y - z), \quad \dot{z} = -\dot{y} - 3z,$$

where the dot denotes differentiation with respect to  $t$ .

Obtain a second order differential equation for  $y$  and hence derive the solution

$$y = Ae^{-t} + Be^{-6t}, \quad z = \frac{1}{2}Ae^{-t} - 2Be^{-6t},$$

where  $A$  and  $B$  are arbitrary constants.

- (i) Obtain the solution that satisfies  $z(0) = 0$  and  $y(0) = 5$ . The quantity of the drug in the brain for this solution is denoted by  $z_1(t)$ .
- (ii) Obtain the solution that satisfies  $z(0) = z(1) = c$ , where  $c$  is a given constant. The quantity of the drug in the brain for this solution is denoted by  $z_2(t)$ .
- (iii) Show that for  $0 \leq t \leq 1$ ,

$$z_2(t) = \sum_{n=-\infty}^0 z_1(t-n),$$

provided  $c$  takes a particular value that you should find.



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