

## STEP III, 2012 Q6

- 6 Let  $x + iy$  be a root of the quadratic equation  $z^2 + pz + 1 = 0$ , where  $p$  is a real number. Show that  $x^2 - y^2 + px + 1 = 0$  and  $(2x + p)y = 0$ . Show further that

$$\text{either } p = -2x \quad \text{or} \quad p = -(x^2 + 1)/x \quad \text{with } x \neq 0.$$

Hence show that the set of points in the Argand diagram that can (as  $p$  varies) represent roots of the quadratic equation consists of the real axis with one point missing and a circle. This set of points is called the *root locus* of the quadratic equation.

Obtain and sketch in the Argand diagram the root locus of the equation

$$pz^2 + z + 1 = 0$$

and the root locus of the equation

$$pz^2 + p^2z + 2 = 0.$$



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](http://NextStepMaths.com)