

STEP III, 2012 Q5

- 5 (i) The point with coordinates (a, b) , where a and b are rational numbers, is called:
 an *integer rational point* if both a and b are integers;
 a *non-integer rational point* if neither a nor b is an integer.
- (a) Write down an integer rational point and a non-integer rational point on the circle $x^2 + y^2 = 1$.
- (b) Write down an integer rational point on the circle $x^2 + y^2 = 2$. Simplify
- $$(\cos \theta + \sqrt{m} \sin \theta)^2 + (\sin \theta - \sqrt{m} \cos \theta)^2$$
- and hence obtain a non-integer rational point on the circle $x^2 + y^2 = 2$.
- (ii) The point with coordinates $(p + \sqrt{2}q, r + \sqrt{2}s)$, where p, q, r and s are rational numbers, is called:
 an *integer 2-rational point* if all of p, q, r and s are integers;
 a *non-integer 2-rational point* if none of p, q, r and s is an integer.
- (a) Write down an integer 2-rational point, and obtain a non-integer 2-rational point, on the circle $x^2 + y^2 = 3$.
- (b) Obtain a non-integer 2-rational point on the circle $x^2 + y^2 = 11$.
- (c) Obtain a non-integer 2-rational point on the hyperbola $x^2 - y^2 = 7$.



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