

STEP III, 2012 Q3 MS

3. The two parabolas, with vertices oriented in the direction of the positive axes, touch in the third quadrant in case (a), and in the first quadrant in the other three cases. In case (b), there are intersections in the second and third quadrants, in (c) in the third and fourth, and in (d), the trickiest case, they are in the third quadrant and in the first, between the touching point and the vertex of the parabola on the x axis. The first result of part (ii) is obtained by eliminating y between the two equations, the second by differentiating and equating gradients, and the third, by eliminating x^4 , ($a^4 = aa^3$), from the first result using the second.

The cases that arise are $a = 1, = 21, \frac{k}{m} = \frac{21}{12} < 2$ (d), $a = \frac{-1+\sqrt{13}}{2}, k = \frac{13}{2}\sqrt{13} - \frac{5}{2}, \frac{k}{m} < 2$ (d), and $a = \frac{-1-\sqrt{13}}{2}, k = \frac{-13}{2}\sqrt{13} - \frac{5}{2}, k < 0$ (a).



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to NextStepMaths.com