

STEP III, 2012 Q3

3 It is given that the two curves

$$y = 4 - x^2 \quad \text{and} \quad mx = k - y^2,$$

where $m > 0$, touch exactly once.

(i) In each of the following four cases, sketch the two curves on a single diagram, noting the coordinates of any intersections with the axes:

- (a) $k < 0$;
- (b) $0 < k < 16$, $k/m < 2$;
- (c) $k > 16$, $k/m > 2$;
- (d) $k > 16$, $k/m < 2$.

(ii) Now set $m = 12$.

Show that the x -coordinate of any point at which the two curves meet satisfies

$$x^4 - 8x^2 + 12x + 16 - k = 0.$$

Let a be the value of x at the point where the curves touch. Show that a satisfies

$$a^3 - 4a + 3 = 0$$

and hence find the three possible values of a .

Derive also the equation

$$k = -4a^2 + 9a + 16.$$

Which of the four sketches in part (i) arise?



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