

STEP III, 2012 Q2 MS

2. The simplification in the opening is $(1 - x^{2^{n+1}})$, obtained by repeated use of the difference of two squares. A simple algebraic rearrangement, followed by taking a limit, the logarithm of both expressions, and differentiation produces the other three results in part (i). Part (ii) can be obtained by replacing x by x^3 in $\ln(1 - x) = -\sum_{r=0}^{\infty} \ln(1 + x^{2^r})$ from part (i), factorising the difference and the sums of cubes and subtracting that part (i) result before differentiating. An alternative is to replicate part (i) using instead the product

$(1 + x + x^2)(1 - x + x^2)(1 - x^2 + x^4)(1 - x^4 + x^8) \dots (1 - x^{2^n} + x^{2^{n+1}})$, but then a little extra care is required with the rearrangement, and consideration of the limit.



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