

## STEP III, 2012 Q13

- 13 (i) The random variable  $Z$  has a Normal distribution with mean 0 and variance 1. Show that the expectation of  $Z$  given that  $a < Z < b$  is

$$\frac{\exp(-\frac{1}{2}a^2) - \exp(-\frac{1}{2}b^2)}{\sqrt{2\pi} (\Phi(b) - \Phi(a))},$$

where  $\Phi$  denotes the cumulative distribution function for  $Z$ .

- (ii) The random variable  $X$  has a Normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Show that

$$E(X | X > 0) = \mu + \sigma E(Z | Z > -\mu/\sigma).$$

Hence, or otherwise, show that the expectation,  $m$ , of  $|X|$  is given by

$$m = \mu(1 - 2\Phi(-\mu/\sigma)) + \sigma\sqrt{2/\pi} \exp(-\frac{1}{2}\mu^2/\sigma^2).$$

Obtain an expression for the variance of  $|X|$  in terms of  $\mu$ ,  $\sigma$  and  $m$ .



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