STEP III, 2012 Q11 MS

11. Various approaches can be used to find the energy terms. If potential energy zero level is taken to be at P, then the initial potential energy is $-2Mg\frac{L}{2}$. When the particle has fallen a distance *x*, the kinetic energy of the particle is $\frac{1}{2}mv^2$, the potential energy of the particle is $-mgx$, the potential energy of the part of the stationary piece of string of length *x* is $-\frac{x}{2L}2Mg\frac{x}{2}$, the potential energy of the remaining piece of (doubled up) string is $-\left(1-\frac{x}{2L}\right)2Mg\left(x+\frac{1}{2}\left(L-\frac{1}{2}x\right)\right)$, and the kinetic energy of the shorter moving piece is $\frac{1}{2}\frac{L-\frac{1}{2}x}{2L}\,2M\,v^2$. This yields the first result and differentiation of it yields the second. As $0<x\leq 2L$, $ML-\frac{Mx}{4}\geq\frac{ML}{2}$, as $Mgx>0$ and the denominator is twice a square $\frac{Mgx\left(mL+ML-\frac{Mx}{4}\right)}{2\left(mL+ML-\frac{xM}{2}\right)^2}>0$, the final result follows.