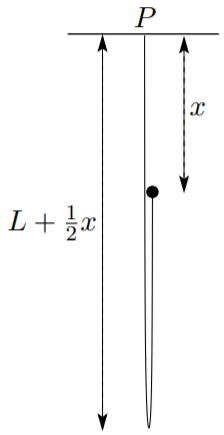
STEP III, 2012 Q11

11 One end of a thin heavy uniform inextensible perfectly flexible rope of length $2L$ and mass $2M$ is attached to a fixed point P. A particle of mass m is attached to the other end. Initially, the particle is held at P and the rope hangs vertically in a loop below P. The particle is then released so that it and a section of the rope (of decreasing length) fall vertically as shown in the diagram.



You may assume that each point on the moving section of the rope falls at the same speed as the particle. Given that energy is conserved, show that, when the particle has fallen a distance x (where $x < 2L$), its speed v is given by

$$v^2 = \frac{2gx\left(mL + ML - \frac{1}{4}Mx\right)}{mL + ML - \frac{1}{2}Mx}.$$

Hence show that the acceleration of the particle is

$$g + \frac{Mgx\left(mL + ML - \frac{1}{4}Mx\right)}{2\left(mL + ML - \frac{1}{2}Mx\right)^2}.$$

Deduce that the acceleration of the particle after it is released is greater than g.