

STEP III, 2012 Q1

- 1 Given that $z = y^n \left(\frac{dy}{dx}\right)^2$, show that

$$\frac{dz}{dx} = y^{n-1} \frac{dy}{dx} \left(n \left(\frac{dy}{dx}\right)^2 + 2y \frac{d^2y}{dx^2} \right).$$

- (i) Use the above result to show that the solution to the equation

$$\left(\frac{dy}{dx}\right)^2 + 2y \frac{d^2y}{dx^2} = \sqrt{y} \quad (y > 0)$$

that satisfies $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$ is $y = \left(\frac{3}{8}x^2 + 1\right)^{\frac{2}{3}}$.

- (ii) Find the solution to the equation

$$\left(\frac{dy}{dx}\right)^2 - y \frac{d^2y}{dx^2} + y^2 = 0$$

that satisfies $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$.



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