STEP III, 2011 Q9 MS

9. For the initial equilibrium position, suppose $\theta = \alpha$, considering potential energy, with potential energy zero level at O, $U = 4mga\cos\theta + 3mga\sin\theta + c$, for equilibrium, $\frac{dU}{d\theta} = 0$, giving $\tan\alpha = \frac{3}{4}$.
Then conserving energy,
$4mga\cos\theta + 3mga\sin\theta + \frac{1}{2}7m\left(a\dot{\theta}\right)^2 = 4mga\cos\alpha + 3mga\sin\alpha$ which having substituted for α gives $7a\left(\dot{\theta}\right)^2 + 8g\cos\theta + 6g\sin\theta = 10g$

(i) Resolving radially in general for Q, if R is the contact force,
$4mg\cos\theta - R = 4ma\dot{\theta}^2$, so when $\theta = \beta$, $R = 0$, and thus $4mg\cos\beta = 4ma\dot{\theta}^2$ and so substituting for $\dot{\theta}^2$ and θ in the energy result gives $15\cos\beta + 6\sin\beta = 10$.

(ii) Resolving tangentially for Q, $4mg\sin\theta - T = 4ma\ddot{\theta}$ and for P,
$T - 3mg\cos\theta = 3ma\ddot{\theta}$ so eliminating $\ddot{\theta}$ between them and re-arranging,
$T = \frac{12}{7}mg(\sin\theta + \cos\theta)$ as required.

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