

STEP III, 2011 Q8

- 8 The complex numbers z and w are related by

$$w = \frac{1 + iz}{i + z}.$$

Let $z = x + iy$ and $w = u + iv$, where x , y , u and v are real. Express u and v in terms of x and y .

- (i) By setting $x = \tan(\theta/2)$, or otherwise, show that if the locus of z is the real axis $y = 0$, $-\infty < x < \infty$, then the locus of w is the circle $u^2 + v^2 = 1$ with one point omitted.
- (ii) Find the locus of w when the locus of z is the line segment $y = 0$, $-1 < x < 1$.
- (iii) Find the locus of w when the locus of z is the line segment $x = 0$, $-1 < y < 1$.
- (iv) Find the locus of w when the locus of z is the line $y = 1$, $-\infty < x < \infty$.



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