

STEP III, 2011 Q7

7 Let

$$T_n = (\sqrt{a+1} + \sqrt{a})^n,$$

where n is a positive integer and a is any given positive integer.

(i) In the case when n is even, show by induction that T_n can be written in the form

$$A_n + B_n\sqrt{a(a+1)},$$

where A_n and B_n are integers (depending on a and n) and $A_n^2 = a(a+1)B_n^2 + 1$.

(ii) In the case when n is odd, show by considering $(\sqrt{a+1} + \sqrt{a})T_m$ where m is even, or otherwise, that T_n can be written in the form

$$C_n\sqrt{a+1} + D_n\sqrt{a},$$

where C_n and D_n are integers (depending on a and n) and $(a+1)C_n^2 = aD_n^2 + 1$.

(iii) Deduce that, for each n , T_n can be written as the sum of the square roots of two consecutive integers.



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