

STEP III, 2011 Q5 MS

5.

$$r^2 d\theta = (x^2 + y^2) \frac{d}{dt} \left(\tan^{-1} \left(\frac{y}{x} \right) \right) dt = (x^2 + y^2) \frac{1}{1 + \left(\frac{y}{x} \right)^2} \frac{\left(x \frac{dy}{dt} - y \frac{dx}{dt} \right)}{x^2} dt = \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right) dt$$

and hence integrating gives the result.

A is $(x - a \cos t, y - a \sin t)$ and B is $(x + b \cos t, y + b \sin t)$

$[A] = \frac{1}{2} \int_0^{2\pi} (x - a \cos t) \left(\frac{dy}{dt} - a \cos t \right) - (y - a \sin t) \left(\frac{dx}{dt} + a \sin t \right) dt$ using (*) which leads directly to $[A] = [P] - af + \pi a^2$.

Replacing $-a$ by b gives $[B] = [P] + bf + \pi b^2$

As $[A] = [B]$, these expressions can be equated to give $f = \pi(a - b)$.

The area between curves C and D is $[A] - [P] = -af + \pi a^2$ which by substitution gives πab as required.



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