

## STEP III, 2011 Q3

- 3 Show that, provided  $q^2 \neq 4p^3$ , the polynomial

$$x^3 - 3px + q \quad (p \neq 0, q \neq 0)$$

can be written in the form

$$a(x - \alpha)^3 + b(x - \beta)^3,$$

where  $\alpha$  and  $\beta$  are the roots of the quadratic equation  $pt^2 - qt + p^2 = 0$ , and  $a$  and  $b$  are constants which you should express in terms of  $\alpha$  and  $\beta$ .

Hence show that one solution of the equation  $x^3 - 24x + 48 = 0$  is

$$x = \frac{2(2 - 2^{\frac{1}{3}})}{1 - 2^{\frac{1}{3}}}$$

and obtain similar expressions for the other two solutions in terms of  $\omega$ , where  $\omega = e^{2\pi i/3}$ .

Find also the roots of  $x^3 - 3px + q = 0$  when  $p = r^2$  and  $q = 2r^3$  for some non-zero constant  $r$ .



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