

STEP III, 2011 Q2

- 2 The polynomial $f(x)$ is defined by

$$f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0,$$

where $n \geq 2$ and the coefficients a_0, \dots, a_{n-1} are integers, with $a_0 \neq 0$. Suppose that the equation $f(x) = 0$ has a rational root p/q , where p and q are integers with no common factor greater than 1, and $q > 0$. By considering $q^{n-1}f(p/q)$, find the value of q and deduce that any rational root of the equation $f(x) = 0$ must be an integer.

- (i) Show that the n th root of 2 is irrational for $n \geq 2$.

- (ii) Show that the cubic equation

$$x^3 - x + 1 = 0$$

has no rational roots.

- (iii) Show that the polynomial equation

$$x^n - 5x + 7 = 0$$

has no rational roots for $n \geq 2$.



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