

STEP III, 2011 Q13 MS

13. (i) $P(X = r) = \binom{k}{r} \left(\frac{b}{n}\right)^r \left(\frac{n-b}{n}\right)^{k-r}$, $P(X = r + 1) = \binom{k}{r+1} \left(\frac{b}{n}\right)^{r+1} \left(\frac{n-b}{n}\right)^{k-r-1}$
 and so $\frac{P(X=r+1)}{P(X=r)} = \frac{k-r}{r+1} \frac{b}{n-b}$. The most probable value of X is the minimum value of r such that $\frac{k-r}{r+1} \frac{b}{n-b} < 1$, because this expression decreases as r increases. All the factors are positive so it is simple to rearrange the algebra to obtain $r > \frac{(k+1)}{n}b - 1$ so $r = \left\lfloor \frac{(k+1)}{n}b \right\rfloor$.
 The answer is not unique when there is a value of r such that $\frac{k-r}{r+1} \frac{b}{n-b} = 1$, in which case, $= \frac{(k+1)}{n}b$, which will only happen if n divides $(k+1)b$.

(ii) Using the same strategy as for part (i), $P(X = r) = \frac{\binom{b}{r} \binom{n-b}{k-r}}{\binom{n}{k}}$,

$$P(X = r + 1) = \frac{\binom{b}{r+1} \binom{n-b}{k-r-1}}{\binom{n}{k}}, \text{ and so } \frac{P(X=r+1)}{P(X=r)} = \frac{k-r}{r+1} \frac{b-r}{n-b-(k-r)+1}.$$

Again, the most probable value of X is the minimum value of r such that

$\frac{k-r}{r+1} \frac{b-r}{n-b-(k-r)+1} < 1$, giving $r = \left\lfloor \frac{(k+1)(b+1)}{(n+2)} \right\rfloor$, and this is not unique if $(n+2)$ divides $(k+1)(b+1)$.



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to NextStepMaths.com