# STEP III, 2011 Q13 MS

13. (i) $P(X=r) = \binom{k}{r}\left(\frac{b}{n}\right)^r\left(\frac{n-b}{n}\right)^{k-r}$ , $P(X=r+1) = \binom{k}{r+1}\left(\frac{b}{n}\right)^{r+1}\left(\frac{n-b}{n}\right)^{k-r-1}$
and so $\frac{P(X=r+1)}{P(X=r)} = \frac{k-r}{r+1}\frac{b}{n-b}$ . The most probable value of $X$ is the minimum value of $r$ such that $\frac{k-r}{r+1}\frac{b}{n-b} < 1$ , because this expression decreases as r increases. All the factors are positive so it is simple to rearrange the algebra to obtain $r > \frac{(k+1)}{n}b - 1$ so $r = \left\lfloor\frac{(k+1)}{n}b\right\rfloor$ .
The answer is not unique when there is a value of r such that $\frac{k-r}{r+1}\frac{b}{n-b} = 1$ , in which case, $= \frac{(k+1)}{n}b$ , which will only happen if $n$ divides $(k+1)b$ .

(ii) Using the same strategy as for part (i) , $P(X=r) = \frac{\binom{b}{r}\binom{n-b}{k-r}}{\binom{n}{k}}$ ,

$P(X=r+1) = \frac{\binom{b}{r+1}\binom{n-b}{k-r-1}}{\binom{n}{k}}$ , and so $\frac{P(X=r+1)}{P(X=r)} = \frac{k-r}{r+1}\ \frac{b-r}{n-b-(k-r)+1}$ .

Again, the most probable value of $X$ is the minimum value of $r$ such that $\frac{k-r}{r+1}\frac{b-r}{n-b-(k-r)+1} < 1$, giving $r = \left\lfloor\frac{(k+1)(b+1)}{(n+2)}\right\rfloor$ , and this is not unique if $(n+2)$ divides $(k+1)(b+1)$.