

STEP III, 2011 Q13

13 In this question, the notation $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x , so for example $\lfloor \pi \rfloor = 3$ and $\lfloor 3 \rfloor = 3$.

- (i) A bag contains n balls, of which b are black. A sample of k balls is drawn, one after another, at random *with* replacement. The random variable X denotes the number of black balls in the sample. By considering

$$\frac{P(X = r + 1)}{P(X = r)},$$

show that, in the case that it is unique, the most probable number of black balls in the sample is

$$\left\lfloor \frac{(k + 1)b}{n} \right\rfloor.$$

Under what circumstances is the answer not unique?

- (ii) A bag contains n balls, of which b are black. A sample of k balls (where $k \leq b$) is drawn, one after another, at random *without* replacement. Find, in the case that it is unique, the most probable number of black balls in the sample.

Under what circumstances is the answer not unique?



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