

## STEP III, 2011 Q12

- 12 The random variable  $N$  takes positive integer values and has pgf (probability generating function)  $G(t)$ . The random variables  $X_i$ , where  $i = 1, 2, 3, \dots$ , are independently and identically distributed, each with pgf  $H(t)$ . The random variables  $X_i$  are also independent of  $N$ . The random variable  $Y$  is defined by

$$Y = \sum_{i=1}^N X_i.$$

Given that the pgf of  $Y$  is  $G(H(t))$ , show that

$$E(Y) = E(N)E(X_i) \quad \text{and} \quad \text{Var}(Y) = \text{Var}(N)(E(X_i))^2 + E(N)\text{Var}(X_i).$$

A fair coin is tossed until a head occurs. The total number of tosses is  $N$ . The coin is then tossed a further  $N$  times and the total number of heads in these  $N$  tosses is  $Y$ . Find in this particular case the pgf of  $Y$ ,  $E(Y)$ ,  $\text{Var}(Y)$  and  $P(Y = r)$ .



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