STEP III, 2011 Q11 MS

11. On the one hand the distance between the point on the disc vertically below $(a, 0,0)$ and P is $b\sin\phi$ as the string length b makes an angle ϕ with the vertical. On the other, it is $2a\sin\frac{1}{2}\theta$, the third side of an isosceles triangle with two radii a at an angle θ, and hence the required result.

The horizontal component of the tension in each string is $T\sin\phi$ and it acts at a perpendicular distance $a\cos\frac{1}{2}\theta$ from the axis of symmetry. Thus the couple is $nT\sin\phi\ a\cos\frac{1}{2}\theta$. Resolving vertically, $nT\cos\phi = mg$. Substituting for T in the expression for the couple and then using $b\sin\phi = 2a\sin\frac{1}{2}\theta$ to eliminate ϕ, gives the required result.

The initial potential energy relative to the position where the strings are vertical is $mgb(1-\cos\phi)$. This is converted into kinetic energy $\frac{1}{2}\frac{1}{2}ma^2\omega^2$. Equating these expressions and once again using $b\sin\phi = 2a\sin\frac{1}{2}\theta$ to eliminate ϕ, gives the required result.