

STEP III, 2011 Q10 MS

10. Suppose Q is displaced x and P is displaced y , and let $\lambda = \frac{1}{2} m a \omega^2$,
then $m\ddot{x} = \frac{\lambda(y-x)}{a}$ and $m\ddot{y} = \frac{-\lambda(y-x)}{a}$.

Adding and integrating leads to $\dot{x} + \dot{y} = u t$.

Subtracting gives $\ddot{y} - \ddot{x} = -\omega^2 (y - x)$ and so $y - x = \frac{u}{\omega} \sin \omega t$ from solving the differential equation and employing the initial conditions that when $t = 0$, $x = y = 0$, $\dot{x} = 0$, and $\dot{y} = u$.

Thus, $x = \frac{1}{2} \left(u t - \frac{u}{\omega} \sin \omega t \right)$ and $y = \frac{1}{2} \left(u t + \frac{u}{\omega} \sin \omega t \right)$. When the string next returns to length a , $y - x = \frac{u}{\omega} \sin \omega t = 0$, $\omega t = \pi$ and so $x = y = \frac{1}{2} \frac{u}{\omega} \pi$ as required. So at this time, $\dot{x} = u$, and $\dot{y} = 0$.

The total time between the impulse and the subsequent collision is $\frac{\pi}{\omega} + \frac{a}{u}$.



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