STEP III, 2011 Q10 MS

10. Suppose Q is displaced x and P is displaced y, and let $\lambda = \frac{1}{2} ma\omega^2$,
then $m\ddot{x} = \frac{\lambda(y-x)}{a}$ and $m\ddot{y} = \frac{-\lambda(y-x)}{a}$.
Adding and integrating leads to $x + y = ut$.
Subtracting gives $\ddot{y} - \ddot{x} = -\omega^2 (y - x)$ and so $y - x = \frac{u}{\omega} \sin \omega t$ from solving the differential equation and employing the initial conditions that when $= 0$, $x = y = 0, \dot{x} = 0$, and $\dot{y} = u$.
Thus, $x = \frac{1}{2}\left(ut - \frac{u}{\omega} \sin \omega\, t\right)$ and $y = \frac{1}{2}\left(ut + \frac{u}{\omega} \sin \omega\, t\right)$. When the string next returns to length a , $y - x = \frac{u}{\omega} \sin \omega\, t = 0$, $\omega\, t = \pi$ and so $x = y = \frac{1}{2}\ \frac{u}{\omega}\pi$ as required.
So at this time, $\dot{x} = u$, and $\dot{y} = 0$.
The total time between the impulse and the subsequent collision is $\frac{\pi}{\omega} + \frac{a}{u}$.