

STEP III, 2010 Q9 MS

9. Resolving radially inwards for the mass P , $mg \sin \theta - R = \frac{mv^2}{a}$,
 where R is the normal reaction of the block on P , and v is the (common) speed of the masses
 when OP makes an angle θ with the table.

Conserving energy, $\frac{1}{2}mv^2 + \frac{1}{2}Mv^2 + mga \sin \theta - Mga\theta = 0$, and making v^2 the subject of
 this formula to substitute in the first equation re-arranged for R ,

$$R = mg \sin \theta - \frac{2mg(M\theta - m \sin \theta)}{m+M} = \frac{mg((3m+M) \sin \theta - 2M\theta)}{m+M} \text{ is found.}$$

Remaining in contact requires this expression to be non-negative for all θ , $0 \leq \theta \leq \frac{\pi}{2}$.

Considering the graphs of $y = a \sin \theta$ and $y = b\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$,

$a \sin \theta - b\theta \geq 0, \forall \theta, 0 \leq \theta \leq \frac{\pi}{2}$ if and only if $a \sin \theta - b\theta \geq 0$ for $\theta = \frac{\pi}{2}$

so $R \geq 0$ for all $\theta, 0 \leq \theta \leq \frac{\pi}{2}$ if and only if $(3m + M) \sin \frac{\pi}{2} - 2M \frac{\pi}{2} \geq 0$ which gives the
 required result.



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