STEP III, 2010 Q9 MS

9. Resolving radially inwards for the mass P, $mg\sin\theta - R = \frac{mv^2}{a}$,
where R is the normal reaction of the block on P, and v is the (common) speed of the masses when OP makes an angle θ with the table.

Conserving energy, $\frac{1}{2}mv^2 + \frac{1}{2}Mv^2 + mga\sin\theta - Mga\theta = 0$, and making v^2 the subject of this formula to substitute in the first equation re-arranged for R,
$R = mg\sin\theta - \frac{2mg(M\theta - m\sin\theta)}{m+M} = \frac{mg((3m+M)\sin\theta - 2M\theta)}{m+M}$ is found.
Remaining in contact requires this expression to be non-negative for all , $0 \leq \theta \leq \frac{\pi}{2}$.
Considering the graphs of $y = a\sin\theta$ and $y = b\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$,
$a\sin\theta - b\theta \geq 0, \forall\theta, 0 \leq \theta \leq \frac{\pi}{2}$ if and only if $a\sin\theta - b\theta \geq 0$ for $\theta = \frac{\pi}{2}$
so $R \geq 0$ for all $\theta, 0 \leq \theta \leq \frac{\pi}{2}$ if and only if $(3m+M)\sin\frac{\pi}{2} - 2M\frac{\pi}{2} \geq 0$ which gives the required result.