

STEP III, 2010 Q7

- 7 Given that $y = \cos(m \arcsin x)$, for $|x| < 1$, prove that

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0.$$

Obtain a similar equation relating $\frac{d^3 y}{dx^3}$, $\frac{d^2 y}{dx^2}$ and $\frac{dy}{dx}$, and a similar equation relating $\frac{d^4 y}{dx^4}$, $\frac{d^3 y}{dx^3}$ and $\frac{d^2 y}{dx^2}$.

Conjecture and prove a relation between $\frac{d^{n+2} y}{dx^{n+2}}$, $\frac{d^{n+1} y}{dx^{n+1}}$ and $\frac{d^n y}{dx^n}$.

Obtain the first three non-zero terms of the Maclaurin series for y . Show that, if m is an even integer, $\cos m\theta$ may be written as a polynomial in $\sin \theta$ beginning

$$1 - \frac{m^2 \sin^2 \theta}{2!} + \frac{m^2(m^2 - 2^2) \sin^4 \theta}{4!} - \dots \quad (|\theta| < \frac{1}{2}\pi)$$

State the degree of the polynomial.



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