

STEP III, 2010 Q6

- 6 The points P , Q and R lie on a sphere of unit radius centred at the origin, O , which is fixed. Initially, P is at $P_0(1, 0, 0)$, Q is at $Q_0(0, 1, 0)$ and R is at $R_0(0, 0, 1)$.
- (i) The sphere is then rotated about the z -axis, so that the line OP turns directly towards the positive y -axis through an angle ϕ . The position of P after this rotation is denoted by P_1 . Write down the coordinates of P_1 .
- (ii) The sphere is now rotated about the line in the x - y plane perpendicular to OP_1 , so that the line OP turns directly towards the positive z -axis through an angle λ . The position of P after this rotation is denoted by P_2 . Find the coordinates of P_2 . Find also the coordinates of the points Q_2 and R_2 , which are the positions of Q and R after the two rotations.
- (iii) The sphere is now rotated for a third time, so that P returns from P_2 to its original position P_0 . During the rotation, P remains in the plane containing P_0 , P_2 and O . Show that the angle of this rotation, θ , satisfies

$$\cos \theta = \cos \phi \cos \lambda,$$

and find a vector in the direction of the axis about which this rotation takes place.



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