STEP III, 2010 Q4

4 **(i)** The number α is a common root of the equations $x^2 + ax + b = 0$ and $x^2 + cx + d = 0$ (that is, α satisfies both equations). Given that $a \neq c$, show that

$$\alpha = -\frac{b-d}{a-c}\,.$$

Hence, or otherwise, show that the equations have at least one common root if and only if

$$(b-d)^2 - a(b-d)(a-c) + b(a-c)^2 = 0\,.$$

Does this result still hold if the condition $a \neq c$ is not imposed?

(ii) Show that the equations $x^2 + ax + b = 0$ and $x^3 + (a+1)x^2 + qx + r = 0$ have at least one common root if and only if

$$(b-r)^2 - a(b-r)(a+b-q) + b(a+b-q)^2 = 0\,.$$

Hence, or otherwise, find the values of b for which the equations $2x^2 + 5x + 2b = 0$ and $2x^3 + 7x^2 + 5x + 1 = 0$ have at least one common root.

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