

STEP III, 2010 Q3 MS

3. The two primitive 4th roots of unity are $\pm i$ so $C_4(x) = (x - i)(x + i) = x^2 + 1$

$$\begin{aligned} C_1(x) &= x - 1, \quad x^2 - 1 = (x - 1)(x + 1) \text{ so } C_2(x) = x + 1, \\ x^3 - 1 &= (x - 1)(x^2 + x + 1) \text{ so } C_3(x) = x^2 + x + 1 \\ x^5 - 1 &= (x - 1)(x^4 + x^3 + x^2 + x + 1) \text{ so } C_5(x) = x^4 + x^3 + x^2 + x + 1 \\ x^6 - 1 &= (x^3 - 1)(x^3 + 1) = (x^3 - 1)(x + 1)(x^2 - x + 1) \text{ so } C_6(x) = x^2 - x + 1 \end{aligned}$$

In part (ii), $C_n(x) = 0 \Rightarrow x^4 = -1 \Rightarrow x^8 = 1$ so n is a multiple of 8, and as there are 4 primitive 8th roots of unity, n must be 8.

$$x^p = 1 \Rightarrow x^p - 1 = 0 \Rightarrow (x - 1)(x^{p-1} + x^{p-2} + x^{p-3} + \dots + 1)$$

1 is the only non-primitive root as no power of any other root less than the p^{th} equals unity, because p is prime, so $C_p(x) = x^{p-1} + x^{p-2} + x^{p-3} + \dots + 1$

No root of $C_n(x) = 0$ is a root of $C_t(x) = 0$ for any $t \neq n$. (For if $t < n$, by the definition of $C_n(x)$, there is no integer t such that $a^t = 1$ when $a^n = 1$. Similarly, if $t > n$.)

Thus if $C_q(x) \equiv C_r(x)C_s(x)$, and if $C_q(x) = 0$, then $C_r(x) = 0$ or $C_s(x) = 0$, so $q = r$ or $q = s$.

If $q = r$, then $C_q(x) \equiv C_r(x)$, and so $C_s(x) \equiv 1$ which is not possible for positive s , and likewise in the alternative case.



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