

STEP III, 2010 Q3

- 3 For any given positive integer n , a number a (which may be complex) is said to be a *primitive n th root of unity* if $a^n = 1$ and there is no integer m such that $0 < m < n$ and $a^m = 1$. Write down the two primitive 4th roots of unity.

Let $C_n(x)$ be the polynomial such that the roots of the equation $C_n(x) = 0$ are the primitive n th roots of unity, the coefficient of the highest power of x is one and the equation has no repeated roots. Show that $C_4(x) = x^2 + 1$.

- (i) Find $C_1(x)$, $C_2(x)$, $C_3(x)$, $C_5(x)$ and $C_6(x)$, giving your answers as unfactorised polynomials.
- (ii) Find the value of n for which $C_n(x) = x^4 + 1$.
- (iii) Given that p is prime, find an expression for $C_p(x)$, giving your answer as an unfactorised polynomial.
- (iv) Prove that there are no positive integers q , r and s such that $C_q(x) \equiv C_r(x)C_s(x)$.



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