

## STEP III, 2010 Q2 MS

2. The expression of  $\cosh a$  in exponentials enables the integral to be written as

$$\int_0^1 \frac{1}{x^2 + x(e^a + e^{-a}) + 1} dx$$

which can in turn can be expressed as

$$\int_0^1 \frac{1}{(x + e^a)(x + e^{-a})} dx$$

and so employing partial fractions this is

$$\frac{1}{(e^a - e^{-a})} \left[ \ln \left( \frac{x + e^{-a}}{x + e^a} \right) \right]_0^1$$

The evaluation of this with simplification of logarithms yields

$$\frac{1}{2 \sinh a} \left( \ln \left( e^a \frac{1 + e^a}{1 + e^{-a}} \right) \right)$$

giving the required result.

In part (ii), the same technique can be employed for both integrals giving, in the first case

$$\begin{aligned} & \int_1^{\infty} \frac{1}{(x + e^a)(x - e^{-a})} dx \\ &= \frac{1}{(e^a + e^{-a})} \left[ \ln \left( \frac{x - e^{-a}}{x + e^a} \right) \right]_1^{\infty} \\ &= \frac{1}{2 \cosh a} \left( a + \ln \left( \coth \frac{a}{2} \right) \right) \end{aligned}$$

and in the second

$$\begin{aligned} & \int_0^{\infty} \frac{1}{(x^2 + e^a)(x^2 + e^{-a})} dx \\ &= \frac{1}{(e^a - e^{-a})} \left[ \frac{1}{e^{-\frac{a}{2}}} \tan^{-1} \left( \frac{x}{e^{-\frac{a}{2}}} \right) - \frac{1}{e^{\frac{a}{2}}} \tan^{-1} \left( \frac{x}{e^{\frac{a}{2}}} \right) \right]_0^{\infty} \\ &= \frac{1}{2 \sinh a} \left( \frac{\pi}{2} 2 \sinh \frac{a}{2} \right) \end{aligned}$$

or alternatively

$$\frac{\pi}{4 \cosh \frac{a}{2}}$$



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