

## STEP III, 2010 Q1 MS

1. The first two parts are obtained by separating off the final term of the summation and expanding the brackets respectively giving  $C = \frac{1}{n+1}(nA + x_{n+1})$ , and

$$B = \frac{1}{n} \sum_{k=1}^n x_k^2 - A^2$$

(the latter given in the question).

By comparison with the expression for  $B$ ,

$$D = \frac{1}{n+1} \sum_{k=1}^{n+1} x_k^2 - C^2$$

which by substituting for

$$\frac{1}{n} \sum_{k=1}^n x_k^2$$

from the expression for  $B$  gives

$$D = \frac{1}{n+1} [n(B + A^2) + x_{n+1}^2] - C^2$$

Substituting for  $C$  from the initial result, the required expression can be obtained which can most neatly be written

$$D = \frac{n}{(n+1)^2} [(n+1)B + (A - x_{n+1})^2]$$

Thus  $(n+1)D = nB + \frac{n}{n+1}(A - x_{n+1})^2$  yielding the first inequality.

Also,  $D - B = \frac{n}{(n+1)^2}(A - x_{n+1})^2 - \frac{1}{n+1}B$  and this quadratic expression is only negative if and only if  $(A - x_{n+1})^2 < \frac{n+1}{n}B$ .

Rearranging the inequality to make  $x_{n+1}$  the subject yields the required result.



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