

## STEP III, 2010 Q13

- 13 In this question,  $\text{Corr}(U, V)$  denotes the product moment correlation coefficient between the random variables  $U$  and  $V$ , defined by

$$\text{Corr}(U, V) \equiv \frac{\text{Cov}(U, V)}{\sqrt{\text{Var}(U)\text{Var}(V)}}.$$

The independent random variables  $Z_1$ ,  $Z_2$  and  $Z_3$  each have expectation 0 and variance 1. What is the value of  $\text{Corr}(Z_1, Z_2)$ ?

Let  $Y_1 = Z_1$  and let

$$Y_2 = \rho_{12}Z_1 + (1 - \rho_{12}^2)^{\frac{1}{2}}Z_2,$$

where  $\rho_{12}$  is a given constant with  $-1 < \rho_{12} < 1$ . Find  $E(Y_2)$ ,  $\text{Var}(Y_2)$  and  $\text{Corr}(Y_1, Y_2)$ .

Now let  $Y_3 = aZ_1 + bZ_2 + cZ_3$ , where  $a$ ,  $b$  and  $c$  are real constants and  $c \geq 0$ . Given that  $E(Y_3) = 0$ ,  $\text{Var}(Y_3) = 1$ ,  $\text{Corr}(Y_1, Y_3) = \rho_{13}$  and  $\text{Corr}(Y_2, Y_3) = \rho_{23}$ , express  $a$ ,  $b$  and  $c$  in terms of  $\rho_{23}$ ,  $\rho_{13}$  and  $\rho_{12}$ .

Given constants  $\mu_i$  and  $\sigma_i$ , for  $i = 1, 2$  and  $3$ , give expressions in terms of the  $Y_i$  for random variables  $X_i$  such that  $E(X_i) = \mu_i$ ,  $\text{Var}(X_i) = \sigma_i^2$  and  $\text{Corr}(X_i, X_j) = \rho_{ij}$ .



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](http://NextStepMaths.com)