

STEP III, 2010 Q12 MS

12. $S - rS = 1 + dr + dr^2 + \dots + dr^n + \dots$ which is 1 plus an infinite GP. Summing that GP and making S the subject produces the displayed result.

$E(A) = 1a + 2(1-a)a + 3(1-a)^2a + \dots + n(1-a)^{n-1}a + \dots$ so making use of the first result with $d = 1$, $r = (1-a)$, $E(A) = a \left\{ \frac{1}{1-(1-a)} + \frac{(1-a)}{(1-(1-a))^2} \right\} = a \left\{ \frac{1}{a} + \frac{1-a}{a^2} \right\} = \frac{1}{a}$

$\alpha = a + (1-a)(1-b)\alpha = a + a'b'a$ or alternatively, $\alpha = a + a'b'a + a'^2b'^2a + \dots$ which both lead to the required result.

$$\beta = 1 - \alpha = \frac{a'b}{1-a'b'} \text{ or alternatively, } \beta = a'b + a'^2b'b + a'^3b'^2b + \dots = \frac{a'b}{1-a'b'}$$

The expected number of shots, S , is given by

$$\begin{aligned} E(S) &= 1a + 2a'b + 3a'b'a + 4a'^2b'b + 5a'^2b'^2a + \dots \\ &= a\{1 + 3a'b' + 5a'^2b'^2 + \dots\} + 2a'b\{1 + 2a'b' + \dots\} \end{aligned}$$

which using the initial result of the question $= a \left[\frac{1}{1-a'b'} + \frac{2a'b'}{(1-a'b')^2} \right] + 2a'b \left[\frac{1}{1-a'b'} + \frac{a'b'}{(1-a'b')^2} \right]$ and can be shown to simplify to the required expression.



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