

STEP III, 2010 Q12

- 12 The infinite series S is given by

$$S = 1 + (1 + d)r + (1 + 2d)r^2 + \cdots + (1 + nd)r^n + \cdots,$$

for $|r| < 1$. By considering $S - rS$, or otherwise, prove that

$$S = \frac{1}{1 - r} + \frac{rd}{(1 - r)^2}.$$

Arthur and Boadicea shoot arrows at a target. The probability that an arrow shot by Arthur hits the target is a ; the probability that an arrow shot by Boadicea hits the target is b . Each shot is independent of all others. Prove that the expected number of shots it takes Arthur to hit the target is $1/a$.

Arthur and Boadicea now have a contest. They take alternate shots, with Arthur going first. The winner is the one who hits the target first. The probability that Arthur wins the contest is α and the probability that Boadicea wins is β . Show that

$$\alpha = \frac{a}{1 - a'b'},$$

where $a' = 1 - a$ and $b' = 1 - b$, and find β .

Show that the expected number of shots in the contest is $\frac{\alpha}{a} + \frac{\beta}{b}$.



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