STEP III, 2010 Q11 MS

11. If the acceleration of the block is a', and the acceleration of the bullet is a'', then
$R - \mu(M+m)g = Ma'$ and $-R = ma''$,
so the relative acceleration $a = a' - a'' = \frac{R}{m} + \frac{R-\mu(M+m)g}{M}$

The initial velocity of the bullet relative to the block is $-u$ and the final velocity of the bullet relative to the block is 0. If the time between the bullet entering the block and stopping moving through the block is T, then using" $v = u + at$ ", $0 = -u + \left(\frac{R}{m} + \frac{R-\mu(M+m)g}{M}\right)T$
For the block, the initial velocity is 0 , the final velocity is v , and again using $v = u + at$,

$v = a'T = \frac{R-\mu(M+m)g}{M}\frac{u}{\left(\frac{R}{m}+\frac{R-\mu(M+m)g}{M}\right)}$ and so
$av = \left(\frac{R}{m} + \frac{R-\mu(M+m)g}{M}\right)\frac{R-\mu(M+m)g}{M}\frac{u}{\left(\frac{R}{m}+\frac{R-\mu(M+m)g}{M}\right)} = \frac{Ru-\mu(M+m)gu}{M}$ as required.
If the distance moved by the block whilst the bullet is moving through the block is s,
using" $v^2 = u^2 + 2as$ ", $v^2 = 2a's$ andso $s = \frac{v^2}{2a'} = \frac{Mv^2}{2(R-\mu(M+m)g)} = \frac{Mv^2}{2\frac{Mav}{u}} = \frac{uv}{2a}$

Once the bullet stops moving through the block, the next initial velocity of block/bullet is v, the final velocity is 0, the acceleration is $-\mu g$, so the distance moved s'using
"$v^2 = u^2 + 2as$" is given by $0 = v^2 - 2\mu g s'$ i.e. $s' = \frac{v^2}{2\mu g}$
Thus the total distance moved is $\frac{uv}{2a} + \frac{v^2}{2\mu g} = \frac{v}{2\mu g a}[\mu g u + av]$

$$= \frac{v}{2\mu g a}\left[\mu g u + \frac{Ru-\mu(M+m)gu}{M}\right]$$

$$= \frac{uv}{2\mu g}\left[\frac{R-\mu m g}{Ma}\right]$$

$$= \frac{uv}{2\mu g}\left[\frac{R-\mu m g}{M}\right]\frac{1}{\frac{R}{m}+\frac{R-\mu(M+m)g}{M}}$$

$$= \frac{uv}{2\mu g}\left[\frac{R-\mu m g}{M}\right]\frac{Mm}{(M+m)(R-\mu m g)} = \frac{muv}{2(M+m)\mu g}$$

If $R < (M+m)\mu g$, then the block does not move, and the bullet penetrates to a depth $\frac{mu^2}{2R}$.