STEP III, 2010 Q11

11 A bullet of mass m is fired horizontally with speed u into a wooden block of mass M at rest on a horizontal surface. The coefficient of friction between the block and the surface is μ. While the bullet is moving through the block, it experiences a constant force of resistance to its motion of magnitude R, where $R > (M+m)\mu g$. The bullet moves horizontally in the block and does not emerge from the other side of the block.

(i) Show that the magnitude, a, of the deceleration of the bullet relative to the block while the bullet is moving through the block is given by

$$a = \frac{R}{m} + \frac{R-(M+m)\mu g}{M}.$$

(ii) Show that the common speed, v, of the block and bullet when the bullet stops moving through the block satisfies

$$av = \frac{Ru-(M+m)\mu g u}{M}.$$

(iii) Obtain an expression, in terms of u, v and a, for the distance moved by the block while the bullet is moving through the block.

(iv) Show that the total distance moved by the block is

$$\frac{muv}{2(M+m)\mu g}.$$

Describe briefly what happens if $R < (M+m)\mu g$.