

STEP III, 2010 Q10 MS

10. Resolving perpendicularly to OB, $ma\ddot{\phi} = -T \cos\left(\frac{\pi}{2} - \theta - \phi\right)$, where the tension in the elastic string is $T = \lambda \frac{PB-c}{c}$. The sine rule $\frac{a}{\sin\theta} = \frac{PB}{\sin\phi}$

Putting these three results together gives the required expression.

Also from the sine rule, $\frac{b}{\sin(\theta+\phi)} = \frac{a}{\sin\theta}$, so for ϕ and θ small, $\frac{b}{\theta+\phi} \approx \frac{a}{\theta}$ yielding the desired result.

From this result, θ may be made the subject of the formula, so that the result

$$ma\ddot{\phi} = -\lambda \left(\frac{a \sin\phi}{c \sin\theta} - 1\right) \sin(\theta + \phi), \text{ which for small angles becomes}$$

$$ma\ddot{\phi} \approx -\lambda \left(\frac{a\phi}{c\theta} - 1\right) (\theta + \phi) \text{ can be written } \ddot{\phi} \approx -\frac{\lambda}{ma} \left(\frac{b-a-c}{c}\right) \left(\frac{b}{b-a}\right) \phi$$

and hence the period is $\tau \approx 2\pi \sqrt{\frac{mac(b-a)}{\lambda b(b-a-c)}}$.



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